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Brief communication

Walsh spectral analysis of binary signals arising from intermittent two-phase flows

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1. Introduction

When two immiscible fluids flow through a pipe, different flow regimes may occur, depending on the flow rate of each individual fluid, on the pipe inclination angle, and on the fluid properties. Air–water flows have been widely studied because they have many applications in chemical, oil, and energy industries. Among the others, a possible flow pattern is represented by the intermittent regime that can be divided into plug flow or elongated bubble flow and slug flow – see Paglianti et al. (1996) and Bertola (2003). Without entering into the details of all possible flow patterns, intermittent flow is characterized by the alternation of large bubbles flowing over a thin liquid film and by liquid slugs, that may contain (or not) small gas bubbles. The distinction between slug and plug flow is an interesting research topic – see for instance Drahoš et al. (1996) – but it is beyond the scope of this paper and, therefore, we will use alternatively the terms slug or intermittent flow.

Intermittent flows are intrinsically unsteady and they have a particular structure, neither periodic in time nor in space, that complicates their investigation. The intermittent behaviour, i.e. the alternating between air and water, causes high pressure and flow rate fluctuations, so that an extremely careful design of the pipeline components is required. Moreover, the low-frequency fluctuations may be in resonance with the characteristic frequency of the pipeline, causing severe damages if not taken into account. The correct prediction of the slug frequency (or alternatively of the slug length) is essential in many practical applications such as the design of gas traps in pipeline facilities.

From another viewpoint, the slug frequency is an input parameter of all the existing slug flow models.

Many efforts have been devoted to the investigation of the statistical properties of intermittent flows, unfortunately most of the correlations is essentially empirical and expresses the averaged slug frequency (or averaged slug length) as a function of the superficial velocities of the two-phases. Attempts to predict theoretically the slug frequency have been carried out. Later, those models have been improved, but closure relations are still needed. A review of slug flow is provided by Fabre and Liné (1992).

In absence of a complete mathematical description of the slug flow, the alternating behaviour must be accounted for by statistical measurements. Considerable amount of work has been carried out to investigate the statistical properties of intermittent flows, see for instance van Hout et al. (2001, 2002, 2003). In the determination of the statistical properties of slug flows, fibre optical probes, impedance or resistive probes are used. Both kind of probes are in some sense binary since they distinguish between water and air, giving the so called phase density function $P_k(\mathbf{x}, t)$

$$P_k(\mathbf{x}, t) = \begin{cases} 1, & \mathbf{x} \in \Omega_k \\ 0, & \mathbf{x} \notin \Omega_k, \end{cases}$$

where Ω_k is the geometrical domain occupied by phase k with $k =$ gas, liquid; $\mathbf{x}$ is the probe location and t is the time. A part of a typical signal is represented in Fig. 1; as can be seen, this is a binary signal and some post-processing is needed to extract the statistical properties of the flow. Rigorously speaking, the signal out of an impedence or an optical probe belongs to a special kind of signals: the so called categorical-valued time series, i.e. it is a signal that distinguishes among a finite set of possible states. Accordingly, the phase density function is a mapping that maps a state (water or air) into a number (0 or 1). Since after the mapping we have a time function defined on $[0, T]$, we look at those post-processing techniques used in time series

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analysis that allow to characterize the alternating between air and water passage.

The easiest technique to describe the alternation between air and water consists in counting the number of slugs per unit time, as proposed by Hubbard (1965), or taking the reciprocal of the mean time delay between two consecutive slugs, as proposed by Ferré (1979). It can be shown that the two definitions are completely equivalent. The result of this post-processing is called average frequency $\bar{f}$; $\bar{f}$ is defined as the number of bubble noses N_s pierced by the probe during the measurement interval T, so that $\bar{f} = N_s/T$. Operatively, on the signal shown in Fig. 1, this technique consists in counting the number of times the signal switches its value from 0 to 1 during the observation window $[0, T]$.

Since the probability distribution function (PDF) of the delay between a nose and the subsequent is not symmetric (see for instance Fig. 1), the mean value is not a good parameter to describe such a distribution. Note that the average of such a distribution is the mean frequency $\bar{f}$ as defined by Ferré (1979). Slug unit duration is computed by evaluating the time between two consecutive switches from 0 to 1 and then the PDF is computed (Fig. 1 gives an example of such a distribution).

Recently, another method has been proposed: it consists in carrying out a Fourier analysis of the signal, see Drahoš et al. (1996). The use of Fourier spectrum to detect the periodicity of a signal was originally proposed by Schuster (1898) who argued that the peak of the power spectral density (PSD) indicates the most important harmonic component f_F of the signal; f_F is not necessarily the same as the mean frequency estimated with the average method. The determination of the most probable frequency, based on the Fourier analysis, corresponds to the decomposition of the signal into a set of sinusoidal and co-sinusoidal waveforms and, implicitly, there is the assumption that the signal is a sum of sinusoidal functions; associated to that, there is the concept of frequency that implies some periodicity within the signal.

Fourier analysis introduces the difference between mean frequency and most probable frequency, but it has the disadvantage to assume somehow a periodic signal made up as a combination of smooth and continuous periodic components. Actually, as already noted by Fabre and Liné (1992) by reducing intermittency to periodicity, the actual very complex flow structure is simplified to an equivalent cell consisting of a long bubble followed by a liquid slug. So, instead of trying to interpret the slug dynamics by means of a basis of periodic functions, a basis of aperiodic functions can provide a better representation. In the present paper, we assess the analysis of the signal by means on an alternative basis: the Walsh functions. Walsh functions are binary, aperiodic, and are classified according to their sequency. The concept of sequency, defined later, characterizes an alternating behaviour without assuming any periodicity.

It is worth noting that out of the signal shown in Fig. 1, it is possible to obtain the PDF concerning the time duration of both bubble and liquid slug. This post-processing approach provides a detailed information on the flow regime, see van Hout et al. (2002). However, the advantage of either Fourier or Walsh analysis is to provide a straightforward way to characterise the flow structure; in addition, the simple models used to compute pressure drops and in situ hold-up require as input a characteristic frequency and not PDFs (either of the bubble or of liquid slug).

Aim of this paper is to introduce to fluid dynamic community the Walsh transform as the proper tool to describe and analyze intermittent flow regimes without assuming any periodicity. Furthermore, the possibility to reliably predict the dynamics of slug arrival is essential in practical applications such as the design of gas traps.

In Section 2, some examples of slug flow signals are analyzed. To those signals, we first apply spectral Fourier analysis and then we apply Walsh spectral analysis and the results are compared quantitatively. Conclusions are given in Section 3. A brief review of Walsh functions and Walsh transform is given in Appendix A.

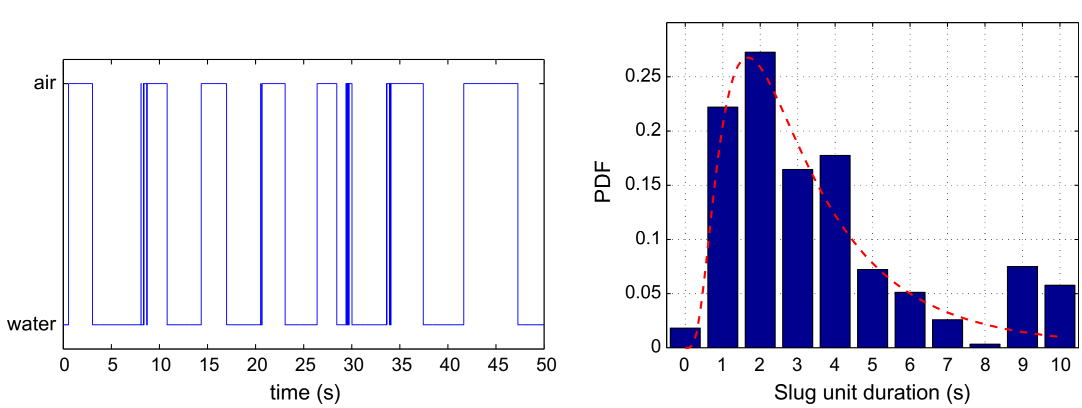

Fig. 1. Example of one-point optical probe signal.

2. Comparison between Fourier and Walsh analyses

As previously mentioned, there are many physical situations in which time series cannot be thought as the superposition of well-separated sinusoids and co-sinusoids. For instance, if the process of interest is discrete or categorical-valued in some finite set (such as square waveforms between 0 and 1) then it makes little sense to correlate the data with smooth sines and cosines. As an alternative, it is suggested that the spectral analysis of time series that contain sharp discontinuities is conducted in the sequency domain by the Walsh transform – see Morettin (1981). An example of Walsh functions is given in Fig. 2. A brief review of Walsh analysis is reported in Appendix A. Note that Walsh functions are not strictly aperiodic, but at certain sequencies they show a periodic behaviour (as the function of order 2, for instance). So Walsh functions are able to catch both periodic and aperiodic dynamics. This seems to be a natural alternative to the usual Fourier analysis since the Walsh transform is based on square-wave Walsh functions. This approach enables to study data in terms of square waves and sequency (switches per unit time) rather than sine waves and frequency (cycles per unit time). Beauchamp (1975) demonstrated that where a signal is derived from a sinusoidally based waveform, Fourier analysis is relevant, while where the signal contains sharp discontinuities and a limited number of levels (as in our case), Walsh analysis is the most appropriate tool.

In this section, we compare Fourier and Walsh analyses for TTL-type signals arising from two-phase intermittent flow. First, the comparison is carried out on a synthetic signal artificially generated to mimic a typical one-point binary probe. As second step, we apply both spectral analysis to two real one-point optical probe signal; in this case, a comparison between Fourier analysis and Walsh analysis is carried out computing the mean square error between the reconstructed signal and the original one.

2.1. Analysis of a synthetic signal

An artificial signal is generated starting from a periodic square wave of frequency 13 and one of frequency 37. The two signals have been multiplied so that the resulting signal is binary and, even if it is originated from two periodic functions, it shows no periodicity at all. Note that, since the signals have values either equal to 0 or to 1, the multiplication is performed by following the dyadic rules – see Beauchamp (1975). So we can not expect that, as occurs for harmonic functions, the product of two functions with frequency f_1 and f_2 will result in a signal corresponding to the sum of two periodic functions with frequencies $f_1 + f_2$ and $f_1 - f_2$. A part of this signal is represented in Fig. 3 for similarity to the real signals that are analysed in the next section; the synthetic signal upon which Fourier and Walsh analyses are performed is made up of 1.8 Msample with a sampling frequency of 2 kHz.

Fourier spectrum is computed by the well-known Welch method. Roughly speaking, this method consists in splitting the signal into shorter windows (actually eight windows), overlapped by 50%, and then computing the spectrum corresponding to each window. To each signal window, a Hamming filter is applied prior to computing the Fourier transform. The Fourier transform is computed by the standard FFT package provided by the software MATLAB©. The obtained spectra are ensemble averaged to compute the spectrum of the whole signal. Also Walsh spectrum was obtained by a similar technique, i.e. splitting the signal into eight windows overlapping by 50% and after Hamming-filtering, the Walsh transform is applied. The

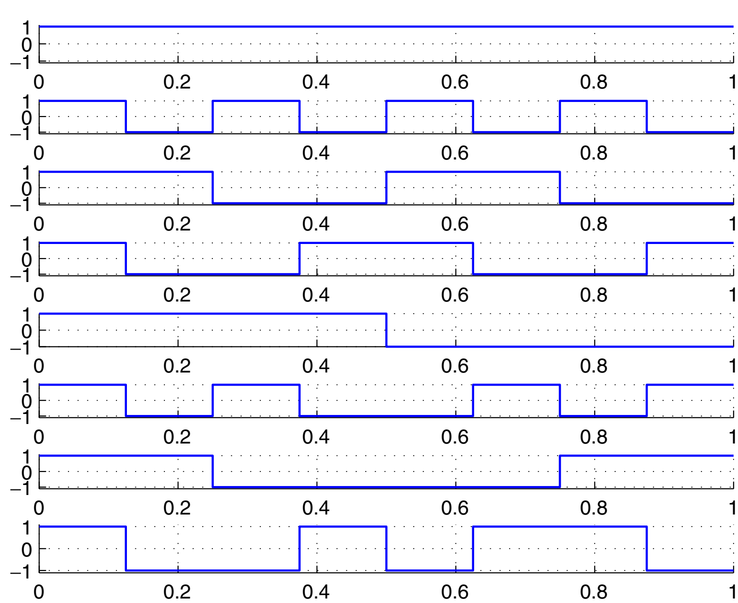

Fig. 2. Walsh functions ordered with Hadamard convention.

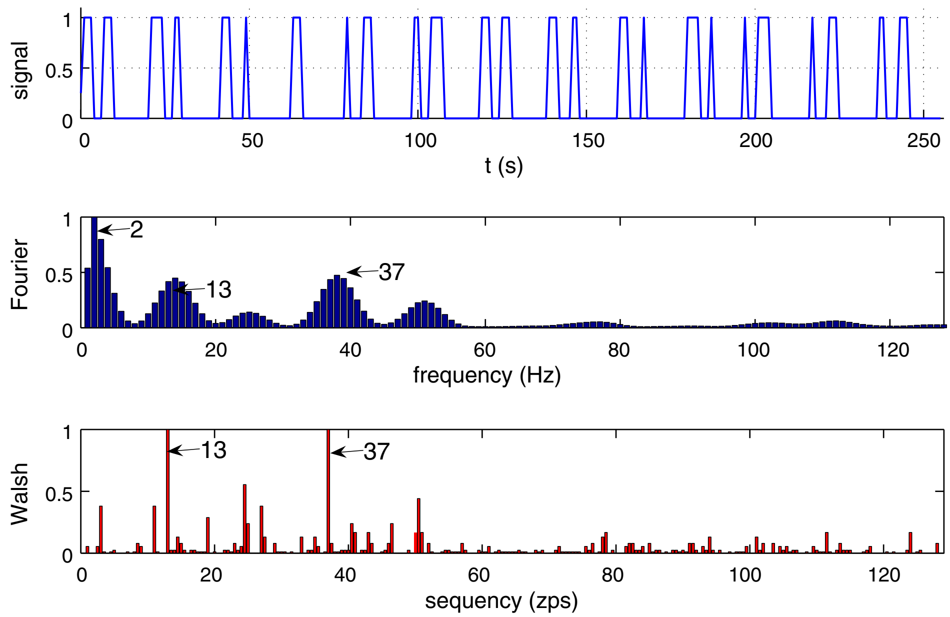

Fig. 3. Comparison between Fourier spectrum and Walsh spectrum for a synthetic signal built upon two square wave between 0 and 1 with frequency 13 and 37. Note that Walsh spectrum clearly points to the two components of the signal, while Fourier analysis predicts a dominant frequency at 2 Hz. Average frequency is at ~0.1 Hz.

ensamble averge of the resulting spectra provides a consistent estimator of the Walsh spectrum for the initial signal, Morettin (1981). The Walsh transform was computed by the so called Fast Walsh Hadamard Transform (FWHT) following the implementation described in Ahmed and Rao (1975).

The average frequency $\bar{f} = N_s/T$ is $\bar{f} = 0.098 \sim 0.1$ Hz. The most probable frequency – f_F – estimated as the frequency corresponding to the peak in the power spectral density (see Fig. 3) is 2 Hz. If one uses the method proposed by Schuster (1898) the results are not correct and lead to a poor modelling of the phenomenon. A second glance at the Fourier spectrum shows that two secondary peaks appear at frequency 13 Hz and 37 Hz. So Fourier analysis detects somehow the right spectral components, however they are not recognized as dominant. Furthermore, from Fourier analysis one is induced to think that the original signal is somehow periodic since the sum of sinusoidal waves (with frequencies 2, 13, and 37) is a periodic function.

Walsh spectrum shows two dominant sequencies (with exactly the same value) at $\lambda_1 = 13$ zps and $\lambda_2 = 37$ zps, indicating the two Walsh functions that have the same zero crossings as the original basic components in addition to that neither Walsh function of the order 13 or of the order 37 are periodic, so one is not induced to interpret the original signal as periodic. Walsh analysis appears as a reliable tool to interpret intermittent, but not periodic, signals as the ones occurring in two-phase slug/plug flows. As wished by Fabre and Liné (1992), Walsh analysis is able to give a consistent representation of an alternating behaviour without reducing it to periodic analysis.

In this case, it can be concluded that Walsh analysis is a more suitable tool to post-process this kind of binary signals. Both average frequency $\bar{f} = 0.1$ Hz and most probable frequency $f_F = 2$ Hz provide a misleading characterization of the signal. For instance, if the final aim of such statistical measurements is to design a gas-trap for a pipeline facility, Fourier analysis would predict two gas slug arrival per second which is, of course, not correct.

2.2. Analysis of a real signal

While for the synthetic signal analysed earlier, the most appropriate post-processing tool can be easily indicated, for a real signal the determination of the best tool is not so trivial. Since the probe signal assumes only values 0 or 1 and it presents sharp discontinuities, Walsh functions seem to be a more suitable basis than sinusoidal and cosinusoidal functions. This observation is, however, subjective and needs to be supported by a quantitative argument. Actually, at the end of this paragraph a more objective way to estimate the goodness of one transform respect to the other is introduced and applied.

Fourier and Walsh spectral analyses are carried out for two signals out of one point optical probe obtained by Arosio and Guilizzoni (2006). These data sets represent the temporal evolution of the phase density function for an air–water flow in a horizontal pipe for two different flow rates. In both cases, the sampling frequency is $f_s = 2$ kHz and the authors collected 1.8 Msamples. The first signal is obtained with air superficial velocity $j_a = 0.3$ m/s and water superficial velocity $j_w = 0.9$ m/s. The second signal is obtained with air superficial velocity $j_a = 1.8$ m/s and

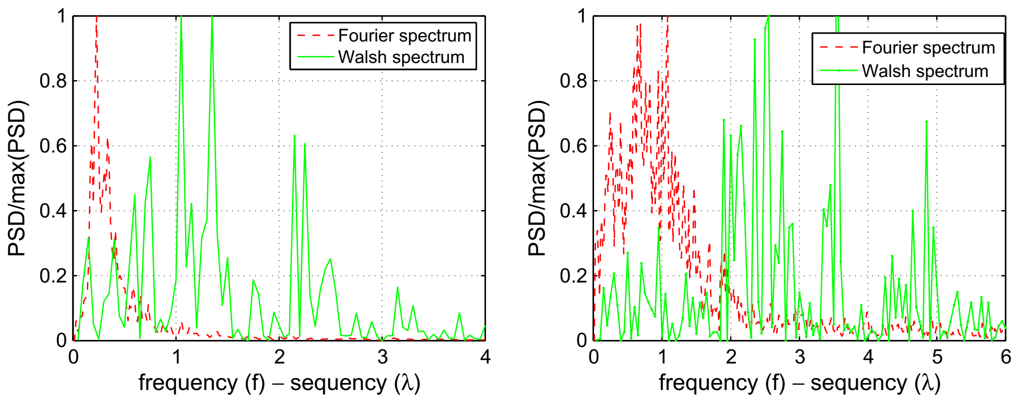

Fig. 4. Fourier and Walsh spectral analysis for a binary optical probe signal, Arosio and Guilizzoni (2006). Left: $j_a = 0.3$ m/s and $j_w = 0.9$ m/s; right: $j_a = 1.8$ m/s and $j_w = 2$ m/s.

water superficial velocity $j_w = 2$ m/s. In both cases the pipe inner diameter is 8 cm.

The computed averaged frequency for the first data set is $\bar{f} = 0.25$ Hz while for the second data set $\bar{f} = 3.24$ Hz. Slug duration PDF is provided in Fig. 1 for the first data set; as expected, such distribution is well-approximated by a log-normal function – see Fig. 1; being the PDF not symmetrical around its mean, the average has little sense.

Fourier and Walsh spectra, shown in Fig. 4, are computed following the procedure described in the previous section.

For the first signal, the dominant frequency f_F is 0.31 Hz while there are two dominant sequencies at $\lambda_1 = 1.1$ zps and at $\lambda_2 = 1.34$ zps, see Fig. 4. It is also evident that while Fourier spectrum shows a broad range of energetic modes around the dominant value, Walsh spectrum shows very clearly some secondary peaks at well defined sequency values. Fourier spectral analysis seems to indicate that most of energy content of the signal is under 1 Hz, while in Walsh analysis peaks are better indicated and spread over a larger sequency range. From a qualitative viewpoint, this result recalls the one obtained for the artificial signal analyzed in the previous paragraph where the dominant frequency is greatly underestimated. Different peaks are present having values of the order of the maximum. Walsh analysis seems to suggest that slug dynamics is much more complex than a simple periodic motion. The resulting dynamics, furthermore, does not show a periodic pattern contrary to what Fourier analysis would suggest.

For the second signal, the dominant frequency f_F is $f_{F,1} = 1.05$ Hz (and a secondary frequency at $f_{F,2} = 0.8$ Hz) while the dominant sequencies are at $\lambda_1 = 2.48$ zps and $\lambda_2 = 3.52$ zps, see Fig. 4. This data set shows a more complex behaviour. Two peaks of comparable amplitude appear in Fourier analysis and the relevant sequencies in Walsh spectrum are more numerous. Both post-processing tools reveal a complex dynamics. From Fourier analysis, it can be seen that the energetic modes are in the range 0–2 Hz, while as in the previous case Walsh analysis is able to better pinpoint the important sequencies.

To quantitatively compare the two tools, the analysis of the MSE as function of the first n significant modes is carried out. This method consists in comparing the mean squared error (MSE) of the signal reconstructed by the first n most energetic modes – $\tilde{x}^{(n)}$ – respect to the original signal x; this method was proposed by Kolmogorov (1941). In other terms, from the spectrum the n most significant modes

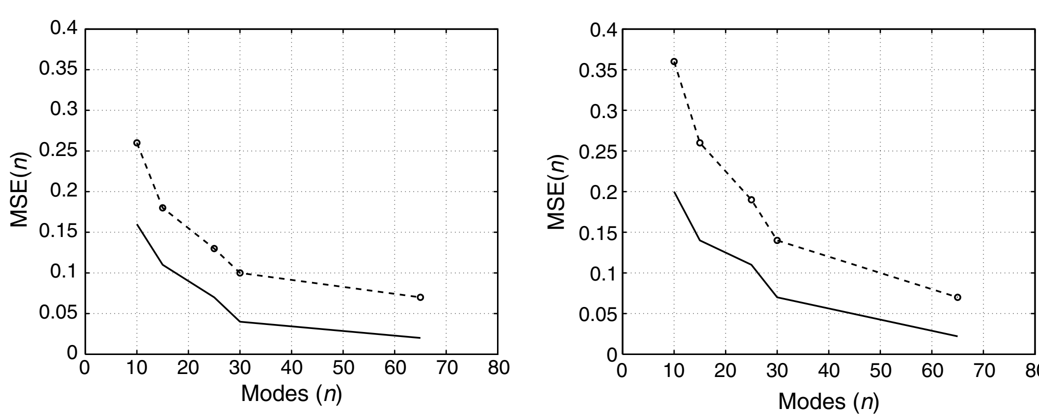

Fig. 5. Mean squared error as function of the number of modes used for the signal reconstruction, see Eq. (1). Continuous line: signal reconstruction by inverse Walsh transform. Dash line: signal reconstruction by inverse Fourier transform. Left: $j_a = 0.3$ m/s and $j_w = 0.9$ m/s; right: $j_a = x1.8$ m/s and $j_w = 2$ m/s.

(most energetic) are chosen and the inverse transform algorithm is applied. The reconstructed signal is compared with the original one, and the mean squared error is computed as

$$\mathrm{MSE} = \frac{1}{N}\sqrt{\sum_{j=0}^{N-1}\left[x_j - \tilde{x}_j^{(n)}\right]^2}, \quad (1)$$

where $\tilde{x}_j^{(n)}$ represents the value of the signal reconstructed upon the first n energetic modes evaluated at instant j. N is the total number of sample. For a given number of modes n, the decomposition that better represents the signal, and hence that it is better suited for its analysis, is the one for which the MSE is lower. Such analysis is summarized in Fig. 5. This figure clearly demonstrates that Walsh functions provide a lower MSE than the Fourier ones for a given value of n. This has to be interpreted that each Walsh function carries more information about the signal than the corresponding Fourier analysis. This conclusion holds both for the case of not aerated liquid slug (Fig. 5-left) and for the case of aerated liquid slug (Fig. 5-right). As expected, increasing the value of n the difference decreases since – being both basis complete – they perfectly match the original signal as $n \to \infty$.

3. Discussion and conclusion

In this paper, a method to characterize the alternating between air and water in intermittent flows without assuming any periodicity is proposed: the Walsh spectral analysis. It has been shown that Walsh analysis can provide a good alternative to Fourier analysis for binary time series. Since Walsh analysis is a mathematically sound method to describe intermittent, but not periodic, signals we believe that, even if Fourier analysis leads to more intuitive results, Walsh functions represent a more natural frame within which interpret intermittent flows. A part from the academic viewpoint, the alternating behaviour appears more efficiently described by the Walsh function also an application viewpoint especially if the aim is to characterize the slug dynamics to correctly design the gas traps. It is evident from both synthetic signals and real ones that the slug arrival is correctly predicted neither by the mean frequency nor by the Fourier spectrum. We have tested Walsh analysis both on artificial signals and on real ones and in both cases, Walsh analysis proved to be more informative than Fourier analysis.

A very common pitfall when using any kind of transform for data analysis is to forget the presence of the analyzing functions in the transformed field and, then, to interpret the features of the analysing functions as characteristics of the phenomena under study. If Fourier analysis is used for the signal proposed here, one has to remember that sines and cosines functions do have a period – and hence have a frequency – while the binary signal is not necessarily periodic. So, Fourier analysis may be misleading in the sense that one is induced to think that is the signal to be periodic. To reduce the risk of a misinterpretation of the phenomena, the analyzing function has been chosen according to the structure of the signal to be analyzed. For the signal out of a binary probe, the elementary object is a square wave and then Walsh functions are closer to that form than Fourier functions.

As final remark, it is worthwhile noting that other methods have been used to characterize the dynamical behaviour of an intermittent flow. Some authors – see Drahoš et al. (1996) – used dynamical pressure time series to extract information on the flow behaviour. The application of diffusional analysis, Giona et al. (1994), and of the methods belonging to the analysis of the deterministic chaos, Drahoš et al. (1996), has been proposed and successfully applied. Even if all those methods look rather different compared to the one proposed here, and compared among themselves, they all have the same spirit: the dynamical characterization of an intermittent flow. Since intermittent flows have a very complex dynamical behaviour, a careful interpretation can be performed only by joining all the possible tools of analysis to tackle the problems from different, but complementary, sides.

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Appendix A. Brief review of Walsh functions and Walsh transform

Rigorously speaking – see Morettin (1981) and Kohn (1980) – the Walsh functions $W(n,t)$, for $n = 1,2,\dots$ and $t \in [0,1)$ are defined as

1. $W(0,t) = 1$ for $t \in [0,1)$,
2. if n, positive integer, has the dyadic expansion $n = \sum_{i=0}^{\infty} x_i 2^i$, with $x_i = 0$ or $x_i = 1$, and $x_i = 0$ for $i > m_r$, then

$$W(n,t) = \prod_{i=0}^{r}\{r_{m_i}(t)\}, \quad (2)$$

where $m_1 \dots m_r$ correspond to the coefficients $x_{m_i} = 1$ and where $\{r_k(t)\}$ are the Rademacher functions, see Morettin (1981).

Walsh functions form an orthonormal basis on [0,1)

$$\int_0^1 W(n,t)W(m,t)\mathrm{d}t = \delta_{nm} \quad (3)$$

furthermore such basis is also complete. While the sinusoids in the Fourier analysis are distinguished by their frequency of oscillation n in terms of the number of complete cycles they make in the interval $0 \leqslant t < 1$, the Walsh functions are distinguished by the number of times n that they switch signs in the unit interval. Since the Walsh functions are aperiodic, the value n in the notation $W(n,t)$ cannot be called frequency as in the case of periodic sinusoid. Harmuth (1969) introduced the term sequency to describe the generalized frequency and to distinguish those functions, such as the Walsh functions, that are not necessarily periodic. Harmuth (1969) noted that the frequency parameter n in the sinusoids – $\sin(2\pi nt)$ – may also be interpreted as one half the number of zero crossings or sign changes per unit time. In analogy to the relationship of frequency to the number of zero crossing or sign changes in periodic functions, Harmuth-sequency is defined has one half the number of zero crossing per unit time – this concept can be applied to aperiodic functions as well as periodic functions; furthermore, Harmuth-sequency coincides with the frequency for a sinusoidal function and, hence, the concept of sequency can be seen as a generalization of the concept of frequency. While frequency is measured in cycles per second (Hz), sequency is measured in zero crossing per second (zps).

If the function $f(t)$ is defined between 0 and 1 with period 1, so that $t \in [0,1)$, and if it is Lebesgue integrable in [0,1) then it can be expanded in Walsh series

$$f(t) \sim \sum_{n=0}^{\infty} a_n W(n,t) \tag{4}$$

with coefficients defined as

$$a_n = \int_0^1 f(t)W(n,t)\mathrm{d}t \quad n = 0, 1, \ldots \tag{5}$$

The Walsh transform of a function $f(t) \in L^2(0,\infty)$ is defined as

$$d(n) = \int_0^{\infty} f(t)W(n,t)\mathrm{d}t. \tag{6}$$

Walsh functions have several properties that are described in detail by Kohn (1980) and Morettin (1981); so the interested reader is suggested to look up these references.

The Walsh transform of $X(0), X(1),\ldots,X(N-1)$ is

$$d_N(\lambda) = N^{-1/2}\sum_{t=0}^{N-1} X(t)W(t,\lambda), \quad 0 \leqslant \lambda < 1. \tag{7}$$

The signal $X(t)$ can be represented as the superposition of Walsh functions at various sequencies,

$$X(t) = \sum_{j=1}^{q} A(j)W(t,\lambda_j), \tag{8}$$

where $A(j)$ are mutually uncorrelated random variables. Eq. (8) is called the inverse discrete Walsh transform.

The Walsh periodogram of data $X(0), X(1),\ldots,X(N-1)$ is defined as

$$I_W(\lambda_j) = \left[N^{-1/2}\sum_{t=0}^{N-1} X(t)W(t,\lambda_j)\right]^2 \tag{9}$$

where λ_j is a sequency of the form $\lambda_j = j/N$, where j indicates the switches per N times points. The Walsh periodogram is essentially the squared correlation of the data with the Walsh functions. Similarly to Fourier analysis, it is possible to plot $I_W(\lambda_j)$ as function of λ_j to inspect for peaks implicitly extending the method proposed by Schuster (1898). The term being squared in Eq. (9) is called Walsh transform of data $X(t)$ and it is indicated as $d_N(\lambda)$. Hence, a consistent estimate of the Walsh spectrum, $f(\lambda)$ is simply the average of the Walsh periodogram, Eq. (9).

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